Course Code: 23HS0830 **R23**

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY: PUTTUR
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QUESTION BANK (DESCRIPTIVE)

Subject with Code: Linear Algebra & Calculus (23HS0830) **Course & Branch**: B.Tech - Common to All

Year & Sem: I-B.Tech & I-Sem **Regulation:** R23

UNIT –I
MATRICES

1	a) Define rank of the matrix.	[L1][CO1]	**[2M]**
	b) Reduce the matrix A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[2M]**
	c) State Cauchy–Binet formulae.	[L1][CO1]	**[2M]**
	d) What is the Consistency and Inconsistency of system of linear equations?	[L1][CO1]	**[2M]**
	e) Solve by Gauss-Seidel method $x - 2y = -3\,; 2x + 25y = 15.$ [Only two iterations]	[L3][CO1]	**[2M]**
2	a) Reduce the matrix A= $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} & \mathbf{5} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} & \mathbf{4} & \mathbf{5} \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[5M]**
	b) Reduce the matrix A to normal form and hence find its rank A= $\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$	[L3][CO1]	**[5M]**
3	a) If A= $\begin{bmatrix} \mathbf{2} & \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{4} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{bmatrix}$ and B= $\begin{bmatrix} \mathbf{1} & \mathbf{6} & \mathbf{0} \\ \mathbf{3} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} \end{bmatrix}$ Verify that $\|\boldsymbol{AB}\|=\|\boldsymbol{A}\|.\|\boldsymbol{B}\|$	[L2][CO1]	**[5M]**
	b) Find whether the following equations are consistent if so solve them $x + y + 2z = 4\,; 2x - y + 3z = 9\,; 3x - y - z = 2.$	[L3][CO1]	**[5M]**
4	a) Reduce the matrix A= $\begin{bmatrix} \mathbf{-2} & \mathbf{-1} & \mathbf{-3} & \mathbf{-1} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{-1} \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[5M]**
	b) Solve completely the system of equations $4x + 2y + z + 3w = 0\,; 6x + 3y + 4z + 7w = 0\,; 2x + y + w = 0.$	[L3][CO1]	**[5M]**
5	Find the inverse of the matrix A= $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ using Gauss-Jordan method.	[L3][CO1]	**[10M]**
6	a) Solve completely the system of equations x+2y+3z=0, 3x+4y+4z=0, 7x+10y+12z=0.	[L3][CO1]	**[5M]**
	b) Show that the equations $x + y + z = 4\,; 2x + 5y - 2z = 3\,; x + 7y - 7z = 5$ are not consistent.	[L2][CO1]	**[5M]**
7	Show that the only real number λ for which the system $x + 2y + 3z = \lambda x\,; 3x + y + 2z = \lambda y\,; 2x + 3y + z = \lambda z$ has non-zero solution is 6. and solve them when λ=6.	[L2][CO1]	**[10M]**
8	Solve the equations $3x + y + 2z = 3\,; 2x - 3y - z = -3\,; x + 2y + z = 4$ Using Gauss elimination method.	[L3][CO1]	**[10M]**

9	Express the following system in matrix form and solve by Gauss elimination method. $2x_1 + x_2 + 2x_3 + x_4 = 6$; $6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$; $4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$; $2x_1 + 2x_2 - x_3 + x_4 = 10$.	[L2][CO1]	**[10M]**
10	Solve the following system of equations by Gauss-Jacobi Iteration method $27x + 6y - z = 85$; $x + y + 54z = 110$; $6x + 15y + 2z = 72$.	[L3][CO1]	**[10M]**
11	Solve the following system of equations by Gauss-Siedel Iteration method $4x + 2y + z = 14$; $x + 5y - z = 10$; $x + y + 8z = 20$.	[L3][CO1]	**[10M]**

UNIT –II

EIGEN VALUES, EIGEN VECTORS AND ORTHOGONAL TRANSFORMATION

1	a) Define Eigen values and Eigen vectors of a matrix.	[L1][CO2]	**[2M]**
	b) Find the Eigne values of the matrix A = $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$	[L3][CO2]	**[2M]**
	c) State Cayley Hamilton theorem	[L1][CO2]	**[2M]**
	d) Convert the symmetric matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ into the quadratic form.	[L2][CO2]	**[2M]**
	e) Find the symmetric matrix corresponding to the quadratic form $ax^2 + 2hxy + by^2$.	[L3][CO2]	**[2M]**
2	a) For the matrix A= $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$.	[L3][CO2]	**[5M]**
	b) Determine the Eigen values of A^{-1} *where* A = $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$	[L3][CO2]	**[5M]**
3	Find the Eigen values and corresponding Eigen vectors of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	[L3][CO2]	**[10M]**
4	Find the Eigen values and corresponding Eigen vectors of the matrix A and also find the eigen values of A^{-1} where A = $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	[L3][CO2]	**[10M]**
5	Determine the modal matrix P of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.Verify that $P^{-1}AP$ is a diagonal matrix.	[L2][CO2]	**[10M]**
6	a) Verify Cayley Hamilton theorem for the matrix A = $\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$.	[L2][CO2]	**[5M]**
	b) Show that the matrix A = $\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ satisfies its characteristic equation.	[L2][CO2]	**[5M]**
7	Verify Cayley Hamilton theorem for A = $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and find A^{-1} *and* A^4 using Cayley Hamilton theorem.	[L3][CO2]	**[10M]**
8	Show that the matrix A = $\begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation. Hence find A^{-1} .	[L2][CO2]	**[10M]**
9	a) State the nature of the Quadratic form $2x_1x_2 + 2x_1x_3 + 2x_2x_3$.	[L1][CO2]	**[5M]**

	b) Identify the nature of the Quadratic form $-3x_1^2 - 3x_2^2 - 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$.	[L2][CO2]	**[5M]**
10	Reduce the Quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into canonical form by Orthogonal transformation and Find the Rank, Index and Signature of the canonical form.	[L3][CO2]	**[10M]**
11	Reduce the Quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2xz - 2yz$ into the canonical form by Orthogonal transformation and discuss its nature.	[L3][CO2]	**[10M]**

UNIT –III
CALCULUS

1	a) State Rolle's theorem.	[L1][CO3]	**[2M]**
	b) Verify the Rolle's Theorem can be applied to the function f(x) = $\tan x$ in [0,π]	[L2][CO3]	**[2M]**
	c) State Lagrange's mean value theorem.	[L1][CO3]	**[2M]**
	d) State Cauchy's mean value theorem.	[L1][CO3]	**[2M]**
	e) Expand Taylor's series of the function f(x) in powers of (x-a).	[L2][CO4]	**[2M]**
2	a) Verify Rolle's Theorem for the function f(x) = $\frac{\sin x}{e^x}$ in $[0, \pi]$	[L2][CO3]	**[5M]**
	b) Verify Lagrange's mean value theorem for f(x) = $\log_e x$ in $[1, e]$.	[L2][CO3]	**[5M]**
3	a) Verify Rolle's Theorem for the function f(x) =$\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a, b]$; a,b>0	[L2][CO3]	**[5M]**
	b) Test whether the Lagrange's Mean value theorem holds f(x)=$x^3 - x^2 - 5x + 3$ in $[0,4]$ and if so find approximate value of c.	[L4][CO3]	**[5M]**
4	a) Verify Rolle's theorem for the function f(x) = x(x+3)$e^{-\frac{x}{2}}$ in [-3,0]	[L2][CO3]	**[5M]**
	b) Verify Cauchy's mean value theorem for f(x) = e^x and g(x) = e^{-x} in [a, b].	[L2][CO3]	**[5M]**
5	a) Show that for any $\mathbf{x > 0}$, $\mathbf{1 + x < e^x < 1 + xe^x}$ using Lagrange's mean value theorem.		
	b) Verify Cauchy's Mean value theorem for $f(x) = x^3$ *and* $g(x) = x^2$ *in* [1,2]	[L2][CO3]	**[5M]**
6	a) Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}(\frac{3}{5}) > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem.	[L2][CO3]	**[5M]**
	b) Verify Cauchy's mean value theorem for $f(x) = sinx$; $g(x) = cosx$ *in* $\left[0, \frac{\pi}{2}\right]$.	[L2][CO3]	**[5M]**
7	a) Express the polynomial $2x^3 + 7x^2 + x$-6 in power of $(x - 2)$ by Taylor's series.	[L3][CO4]	**[5M]**
	b) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ up to the term containing $\left(x- \frac{\pi}{2}\right)^4$ assigning Taylor's series.	[L2][CO4]	**[5M]**
8	a) Expand $\log_e x$ in powers of (x-1) and hence evaluate $\log 1.1$ correct to 4 decimal places using Taylor's theorem.	[L2][CO4]	**[5M]**
	b) Obtain the Maclaurin's series expression of the following functions: i) e^x ii) $\cos x$ iii) $\sin x$	[L2][CO4]	**[5M]**
9	Verify Taylor's theorem for $f(x) = (1 - x)^{\frac{5}{2}}$ with Lagrange's form of remainder up to 2 terms in the interval [0,1].	[L2][CO4]	**[10M]**
10	a) Calculate the approximate value of $\sqrt{10}$ correct to 4 decimal places using Taylor's theorem.	[L3][CO4]	**[5M]**
	b) Show that $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ by Maclaurin's theorem.	[L2][CO4]	**[5M]**
11	Using Maclaurin's series expand $\tan x$ up to the fifth power of x and hence find the series for log (sec x).	[L3][CO4]	**[10M]**

UNIT –IV

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PARTIAL DIFFERENTIATION AND APPLICATIONS
(MULTI VARIABLE CALCULUS)

1	a) Define Continuity of a function of two variables at a point.	[L1][CO5]	**[2M]**
	b) Evaluate $\lim_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1}$.	[L5][CO5]	**[2M]**
	c) If $x = u(1-v); y = uv$ $then\ prove\ that$ $J\left(\frac{x,y}{u,v}\right) = u$	[L2][CO5]	**[2M]**
	d) State Functional Dependence.	[L1][CO5]	**[2M]**
	e) Define Extreme value of a function of two variables.	[L1][CO5]	**[2M]**
2	a) If $U = log(x^3+y^3+z^3-3xyz)$, $prove\ that$ $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 U = \frac{-9}{(X+Y+Z)^2}$	[L5][CO5]	**[5M]**
	b) If $u = tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$ then Prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$.	[L5][CO5]	**[5M]**
3	a) $u = sin^{-1}(x-y), where\ x = 3t, y = 4t^3$, $then\ show\ that$ $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$ $by\ total\ derivative.$	[L2][CO5]	**[5M]**
	b) If $u = f(y-z, z-x, x-y)$ prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$ by using Chain rule .	[L3][CO5]	**[5M]**
5	Expand $x^2y+3y-2$ in powers of $(x-2)$ and $(y+2)$ up to the term of 3rd degree.	[L2][CO5]	**[10M]**
6	a) Expand $e^x siny$ in powers of x and y by Maclaurin series.	[L2][CO5]	**[5M]**
	b) If $u = x^2-2y;\ v = x+y+z,\ w = x-2y+3z$,then find Jacobian $J\left(\frac{u,v,w}{x,y,z}\right)$.	[L1][CO5]	**[5M]**
7	a) If $\boldsymbol{u = \frac{x+y}{1-xy}}$ $\boldsymbol{and\ v = tan^{-1}x + tan^{-1}y}$, $\boldsymbol{find}$ $\frac{\partial(u,v)}{\partial(x,y)}$?	[L1][CO5]	**[5M]**
	b) Verify if $\boldsymbol{u = 2x-y+3z}$, $\boldsymbol{v = 2x-y-z}$, $\boldsymbol{w = 2x-y+z}$ are functionally dependent and if so, find the relation between them.	[L5][CO5]	**[5M]**
8	Examine the maxima and minima, if any, of the function $f(x) = x^3y^2(1-x-y)$.	[L4][CO5]	**[10M]**
9	a) Examine the function for extreme value $\mathbf{f(x,y) = x^4+y^4-2x^2+4xy-2y^2}$; (x>0,y>0).	[L4][CO5]	**[5M]**
	b) Find the minimum value of $x^2+y^2+z^2$ $given$ $x+y+z = 3a$.	[L1][CO5]	**[5M]**
10	a) Find the stationary points of $u(x,y) = sinx.siny.sin(x+y)$ $where$ $0 < x < \pi, 0 < y < \pi$ and find the maximum of u.	[L1][CO5]	**[5M]**
	b) Find the shortest distance from origin to the surface $xyz^2 = 2$.	[L1][CO5]	**[5M]**
11	a) Find a point on the plane $\mathbf{3x+2y+z-12 = 0}$, which is nearest to the origin.	[L1][CO5]	**[5M]**
	b) Find the points on the sphere $x^2+y^2+z^2 = 4$ that are closest and farthest from the point (3, 1,-1).	[L1][CO5]	**[5M]**

UNIT –V

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R23

MULTIPLE INTEGRALS
(MULTI VARIABLE CALCULUS)

1	a) Evaluate $\int_0^2 \int_0^x y\, dy\, dx$	[L5][CO6]	**[2M]**
	b) Evaluate $\int_0^\pi \int_0^{a\sin\theta} r\, dr\, d\theta$	[L5][CO6]	**[2M]**
	c) Transform the integral into polar coordinates, $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\, dy\, dx$.	[L2][CO6]	**[2M]**
	d) Find the area enclosed by the parabolas $x^2 = y\ and\ y^2 = x$.	[L1][CO6]	**[2M]**
	e) Evaluate $I = \int_0^1 \int_1^2 \int_2^3 xyz\, dx\, dy\, dz$.	[L5][CO6]	**[2M]**
2	a) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)dxdy$	[L5][CO6]	**[5M]**
	b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$	[L5][CO6]	**[5M]**
3	a) Evaluate $\iint (x^2+y^2)dxdy$ in the positive quadrant for which $x+y \le 1$.	[L5][CO6]	**[5M]**
	b) Evaluate $\int_0^{log2} \int_0^x \int_0^{x+y} e^{x+y+z}\, dzdydx$.	[L5][CO6]	**[5M]**
4	a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dydx$	[L5][CO6]	**[5M]**
	b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$ by converting to polar coordinates.	[L5][CO6]	**[5M]**
5	a) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.	[L2][CO6]	**[5M]**
	b) Evaluate the integral by transforming into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\,dxdy$.	[L3][CO6]	**[5M]**
6	a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)dxdy$.	[L5][CO6]	**[5M]**
	b) Evaluate the integral by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}dydx$.	[L5][CO6]	**[5M]**
7	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} (xy)dydx$ and hence evaluate the same.	[L1][CO6]	**[10M]**
8	a) By changing order of integration, evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$.	[L3][CO6]	**[5M]**
	b) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)dxdydz$	[L5][CO6]	**[5M]**
9	a) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	[L1][CO6]	**[5M]**
	b) Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\, dzdxdy$.	[L5][CO6]	**[5M]**
10	a) Find the volume common to the cylinders $x^2+y^2 = a^2\ and\ x^2+z^2 = a^2$.	[L1][CO6]	**[5M]**
	b) Evaluate $\int\int\int (x^2+y^2+z^2)\, dx\, dy\, dz$ taken over the volume enclosed by the sphere $x^2+y^2+z^2 = 1$, by transforming into spherical polar coordinates.	[L5][CO6]	**[5M]**
11	a) Evaluate the triple integral $\iiint xy^2zdxdydz$ taken through the positive octant of the sphere $x^2+y^2+z^2 = a^2$.	[L5][CO6]	**[5M]**
	b) Calculate the volume of the solid bounded by the planes $x = 0, y = 0, x+y+z = a\ and\ z = 0$	[L1][CO6]	**[5M]**

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